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Bethe-Salpeter Approach for the P_{33} Elastic Pion-Nucleon Scattering in Heavy Baryon Chiral Perturbation Theory.

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Abstract

Heavy Baryon Chiral Perturbation Theory (HBChPT) to leading order provides a kernel to solve the Bethe-Salpeter equation for the P_{33} ($\Delta(1232)$-channel) $\pi - N$ system, in the infinite nucleon mass limit. Crossed Born terms include, when iterated within the Bethe-Salpeter equation, both *all* one- and *some* two-pion intermediate states, hence preserving elastic unitarity below the two-pion production threshold. This suggests searching for a solution with the help of dispersion relations and suitable subtraction constants, when all in-elasticities are explicitly neglected. The solution allows for a successful description of the experimental phase shift from threshold up to $\sqrt{s} = 1500$ MeV in terms of four subtraction constants. Next-to-leading order HBChPT calculations are also used to estimate the unknown subtraction constants which appear in the solution. Large discrepancies are encountered which can be traced to the slow convergence rate of HBChPT.

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1 Introduction

The study of meson-baryon scattering is one of the most challenging topics of particle physics at low and intermediate energies. Firstly, because there exist accurate experimental data and secondly because it provides an excellent scenario where Chiral Symmetry Breaking (ChSB) can be tested. Unlike the light pseudo-scalar meson–meson scattering, the role played by ChSB in the pseudo-scalar meson–baryon scattering case turns out to be more subtle and the construction of an Effective Field Theory (EFT) to describe the latter processes becomes more cumbersome. In the light meson–meson sector the expansion parameter is $p^2/(4\pi f)^2$, being p^μ the four-momentum of the Goldstone bosons [1] and f the corresponding meson weak decay constant. Since the baryon mass is already of the order of $4\pi f$, such an expansion does not work in practice for the meson–baryon sector, though some efforts were undertaken [2]. This drawback was overcome by formulating a new consistent derivative expansion for baryons in a chiral effective field theory [3, 6, 8], called Heavy Baryon Chiral Perturbation Theory (HBChPT). HBChPT is a reformulation of Chiral Perturbation Theory (ChPT) with baryons where the baryon mass is shifted from the propagator to the interaction couplings of the EFT [3]. In this way higher orders are suppressed either by powers of $p^2/(4\pi f)^2$ or by powers of p/M, where M is the baryon mass and p is either the light pseudo-scalar meson mass or the off-shellness of the baryon, respectively. Recently, a fully relativistic formulation consistent with the power counting proposed in HBChPT has been proposed [5], but there is so far a lack of practical applications.

Standard HBChPT has been applied to the πN system several times already in subsequently higher orders [8, 9, 10, 11] and successful fits in the close to threshold region have been found. On the other hand HBChPT, as it is also the case for ChPT in the meson sector, is unable to describe resonances, which clearly indicate the appearance of non-perturbative chiral dynamics. Resonances, play a crucial role in meson-baryon systems. For instance, the low and intermediate energy πN scattering is mostly dominated by the $\Delta(1232)$ resonance, since it is close to threshold and its coupling to pions and nucleons turns out to be quite strong [12]. Hence, the Δ degrees of freedom are often explicitly incorporated in any effective approach aiming at describing low and intermediate energy pion-nucleon dynamics to some degree of accuracy [7]-[13].

Exact Unitarity plays a crucial role in the description of resonances, since at the corresponding energy the scattering amplitude takes its maximum possible value. HBChPT only restores unitarity perturbatively and hence even the computation of phase-shifts becomes ambiguous beyond perturbation theory. Actually, in previous works the unitarization program of HBChPT for the lowest S and P partial wave amplitudes obtained in has already been studied up to third order in the chiral expansion, by using the Inverse Amplitude Method (IAM) either in its most straightforward version [14] or within an improved IAM [15][4]. We note here that since the unitarity correction takes first place at

[3]This program was firstly suggested, consistently formulated and experimentally tested in the case of heavy quark ($Q = b$, c) physics [4].

[4]The literature on unitarization methods is vast. References in the present context can be traced from Ref. [15].

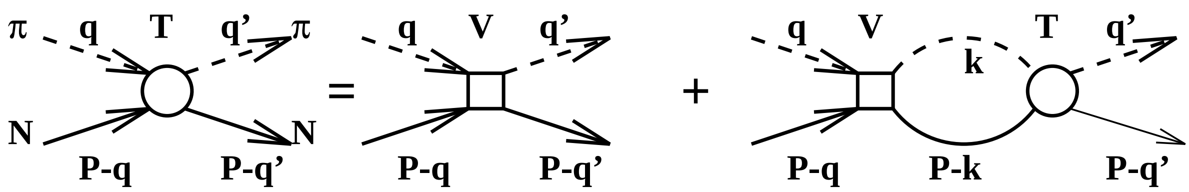

Figure 1: Diagrammatic representation of the BSE equation. It is also sketched the used kinematics.

NNLO in HBChPT one needs to go to that order in order to unitarize à la IAM.

The aim of the present work is to show how a unitarization framework based on the Bethe-Salpeter Equation (BSE) [16] and HBChPT to lowest order yields to a very simple expression for the scattering amplitude which preserves unitarity and provides a satisfactory description of πN elastic scattering data. Our work can be regarded as a natural extension of the approach previously proposed in Refs. [17] and [18] for $\pi\pi$ scattering to the physics of the πN system.

We have concentrated our attention into the P_{33} (L_{2I2J}) channel, and show that the existence of the Δ resonance can be understood in terms of the πN interactions constrained to HBChPT. Hence there is no particular need to include the Δ-degrees of freedom explicitly in order to provide a successful description of the experimentally observed phase-shifts. The extension to other partial waves as well as higher order contributions in HBChPT will be analyzed and presented elsewhere [19].

2 The Bethe-Salpeter Equation and the Elastic P_{33} πN Amplitude in HBChPT.

The BSE for the elastic πN scattering amplitude (T^I) in a given isospin channel ,I, reads

$$t^I_P(q,q') = v^I_P(q,q') + \mathrm{i}\int \frac{d^4k}{(2\pi)^4} t^I_P(k,q')D(k)S(P-k)v^I_P(q,k) \tag{1}$$

with

$$\begin{aligned} T^I_P(q,q')_{\sigma,\sigma'} &= \bar{u}_{\sigma'}(P-q')\, t^I_P(q,q')\, u_\sigma(P-q) \\ V^I_P(q,q')_{\sigma,\sigma'} &= \bar{u}_{\sigma'}(P-q')\, v^I_P(q,q')\, u_\sigma(P-q) \end{aligned} \tag{2}$$

where the kinematical variables are given in Fig. 1 and V^I, D and S are the two particle irreducible amputated Green function, pion and nucleon propagators respectively. t^I, v^I and u are matrices and a spinor [5] in the nucleon Dirac space and finally, σ, σ' are nucleon spin indices (helicity, covariant spin, etc...). The normalization of the amplitude T^I is determined by its relation with the center or mass (CM) differential cross section, in the isospin channel I, and it is given by

$$\frac{d\sigma}{d\Omega}\left(\vec{q}', \sigma' \leftarrow \vec{q}, \sigma\right) = \frac{1}{64\pi^2 s}\left| T^I_P(q,q')_{\sigma\sigma'}\right|^2 \tag{3}$$

[5] We use the normalization $\bar{u}u = 2M$.

where $s = P^2$. Rotational, parity and time reversal invariances ensures for the on shell particles

$$T_P^I(q,q')_{\sigma,\sigma'} = -8\pi\sqrt{s}\left\{\mathcal{A}^I(s,\theta)\delta_{\sigma\sigma'} + \mathrm{i}\,\mathcal{B}^I(s,\theta)\,(\hat{n}\cdot\vec{\sigma})_{\sigma'\sigma}\right\} \tag{4}$$

with θ the CM angle between the initial and final pion three momentum and $\hat{n}$ a unit three-vector orthogonal to $\vec{q}$ and $\vec{q}'$. Partial waves, $f_{IL}^J(s)$, are related to $\mathcal{A},\mathcal{B}$ by

$$\begin{aligned}
\mathcal{A}^I(s,\theta) &= \sum_L \left[(L+1)\; f_{IL}^{L+\frac{1}{2}}(s) + L\; f_{IL}^{L-\frac{1}{2}}(s)\right] P_L(\cos\theta) \\
\mathcal{B}^I(s,\theta) &= -\sum_L \left[f_{IL}^{L+\frac{1}{2}}(s) - \; f_{IL}^{L-\frac{1}{2}}(s)\right] \frac{dP_L(\cos\theta)}{d\theta}
\end{aligned} \tag{5}$$

The phase of the amplitude T^I is such that the relation between the partial wave amplitudes and phase-shifts is the usual one, $f_{IL}^J(s) = e^{\mathrm{i}\,\delta_{IL}^J(s)}\sin\delta_{IL}^J(s)/|\,\vec{q}\,|_{CM}$. Hence, on-shell unitarity implies for $s \geq (m+M)^2$

$$\mathrm{Im}[f_{IL}^J(s)]^{-1} = -|\,\vec{q}\,|_{CM} = -\frac{\lambda^{\frac{1}{2}}(s,M^2,m^2)}{2\sqrt{s}} \tag{6}$$

with $M = 938.27$ MeV, $m = 139.57$ MeV, the nucleon and pion masses respectively, and $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz$.

Note that, to solve Eq. (1) both the off-shell *potential* and amplitude are required. Clearly, for the exact *potential* V^I and propagators D and S, the BSE provides an exact solution of the scattering amplitude T^I [16]. Obviously an exact solution for T^I is not accessible, since V^I, D and S are not exactly known. We propose an expansion along the lines of HBChPT both for the exact *potential* (V^I) and the exact propagators.

The lowest order of the HBChPT expansion in the P_{33} channel leads to [9, 10]

$$\begin{aligned}
V_P^{I=\frac{3}{2}}(q,q')_{\sigma,\sigma'} &\approx V_P^{I=\frac{3}{2}}(q,q')_{\sigma,\sigma'}|_{\rm crossed} + V_P^{I=\frac{3}{2}}(q,q')_{\sigma,\sigma'}|_{\rm contact} \\
V_P^{I=\frac{3}{2}}(q,q')_{\sigma,\sigma'}|_{\rm crossed} &= 2\left(\frac{g_A}{f}\right)^2 \bar{u}_{\sigma'}(Mv+p')\,\frac{S\cdot q\; S\cdot q'}{v\cdot(p-q')+\mathrm{i}\epsilon} u_\sigma(Mv+p) \\
V_P^{I=\frac{3}{2}}(q,q')_{\sigma,\sigma'}|_{\rm contact} &= \frac{1}{4f^2}\,\bar{u}_{\sigma'}(Mv+p')\,(v\cdot q + v\cdot q')\,u_\sigma(Mv+p) \\
D(k) &\approx \frac{1}{k^2-m^2+\mathrm{i}\epsilon} \qquad S(p) \approx \frac{1}{v\cdot p+\mathrm{i}\epsilon}
\end{aligned} \tag{7}$$

where the velocity v is a time-like unit four-vector, in terms of which the nucleon momenta can be written as $P - q\,(') = Mv + p\,(')$ with $v\cdot p\,(') << M$. The EFT can be written in terms of nucleon fields $N_v(x)$ with a definite velocity v^μ which are related to the original nucleon fields $\Psi(x)$ by $N_v(x) = e^{\mathrm{i}\,M\,\not{v}\,v\cdot x}\Psi(x)$. On the other hand, $S_\mu = \frac{\mathrm{i}}{2}\gamma_5\sigma_{\mu\nu}v^\nu$ is defined

in terms of the velocity and the Dirac matrices[6]. Finally, $f = 92.4$ MeV and $g_A = 1.26$ are the pion weak decay constant and the vector axial coupling, respectively. Note that because of isospin conservation, the direct Born term does not contribute to the $I = \frac{3}{2}$ *potential*. Likewise due to angular momentum conservation the contact term does not contribute either to the P_{33} πN scattering and we will ignore it from now on. Thus at lowest order, the BSE (Eq. (1)) for the P_{33} channel reads:

$$
\begin{aligned}
t_P^{I=\frac{3}{2}}(q,q') &= 2\left(\frac{g_A}{f}\right)^2 \frac{S\cdot q\; S\cdot q'}{v\cdot(p-q')+\mathrm{i}\epsilon} + 2\mathrm{i}\left(\frac{g_A}{f}\right)^2 \int \frac{d^4k}{(2\pi)^4}\, t_P^{I=\frac{3}{2}}(k,q') \times \\
&\qquad \frac{1}{k^2-m^2+\mathrm{i}\epsilon}\, \frac{1}{v\cdot P - M - v\cdot k+\mathrm{i}\epsilon}\, \frac{S\cdot q\; S\cdot k}{v\cdot P - M - v\cdot(k+q)+\mathrm{i}\epsilon}
\end{aligned} \tag{9}
$$

We look for solutions to the above equation, suggested by a one-loop calculation, of the form

$$
t_P^{I=\frac{3}{2}}(q,q') = A(v\cdot q, v\cdot q'\,)\, S\cdot q\, S\cdot q' + B(v\cdot q, v\cdot q'\,)\, S\cdot q'\, S\cdot q \tag{10}
$$

where A, B are functions of two variables to be determined. Note that, as a simple one loop calculation shows, there appears a new dependence $(S\cdot q'\, S\cdot q)$ not present in the lowest order *potential* $V^{I=\frac{3}{2}}$. That is similar to what happens in standard HBChPT [9, 10]. The Eq. (9) determines the functions A, B, which turn out to satisfy the following integral equations

$$
\begin{aligned}
A(v\cdot q, v\cdot q'\,) &= 2\left(\frac{g_A}{f}\right)^2 \frac{1}{v\cdot P - M - v\cdot(q+q')+\mathrm{i}\epsilon} - \frac{1}{3}\left(\frac{g_A}{f}\right)^2 \mathrm{i}\int \frac{d^4k}{(2\pi)^4}\frac{k^2-(v\cdot k)^2}{k^2-m^2+\mathrm{i}\epsilon} \\
&\times \quad \frac{1}{v\cdot P - M - v\cdot k+\mathrm{i}\epsilon}\, \frac{A(v\cdot k, v\cdot q'\,)}{v\cdot P - M - v\cdot(k+q)+\mathrm{i}\epsilon}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
B(v\cdot q, v\cdot q'\,) &= -\frac{1}{6}\left(\frac{g_A}{f}\right)^2 \mathrm{i}\int \frac{d^4k}{(2\pi)^4}\frac{k^2-(v\cdot k)^2}{k^2-m^2+\mathrm{i}\epsilon}\, \frac{1}{v\cdot P - M - v\cdot k+\mathrm{i}\epsilon} \\
&\times \quad \frac{A(v\cdot k, v\cdot q'\,) - B(v\cdot k, v\cdot q'\,)}{v\cdot P - M - v\cdot(k+q)+\mathrm{i}\epsilon}
\end{aligned} \tag{12}
$$

The above set of linearly coupled integral equations needs to be regularized, solved and renormalized. This program is highly non-trivial, and hence to draw solutions to these equations we will make use of the analytical structure of the involved functions.

[6] The following properties are satisfied [7]

$$
S\cdot v = 0 \quad S^2 = -\frac{3}{4} \qquad \{S_\mu, S_\nu\} = -\frac{1}{2}\left(g_{\mu\nu} - v_\mu v_\nu\right) \qquad [S_\mu, S_\nu] = \mathrm{i}\,\epsilon_{\mu\nu\rho\alpha} v^\rho S^\alpha \tag{8}
$$

with $\epsilon_{0123} = +1$.

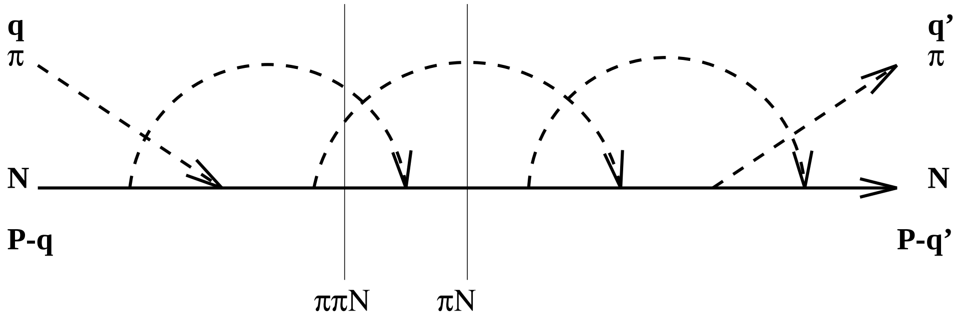

Figure 2: Diagrammatic representation of the BSE equation obtained by iterating only the crossed term potential, defined in Eq. (7). Vertical lines represent the $\pi N \to \pi N$ and $\pi N \to \pi\pi N$ cuts.

3 Elastic Unitarity and On-Shell Dispersion Relations.

For on-shell scattering and at leading order in the $1/M$ expansion, implicit in HBChPT formalism, LAB and CM systems coincide. Then, choosing $v^\mu = (1, \vec{0})$ and taking into account that in the infinitely heavy nucleon limit $v\cdot q = v\cdot q' = v\cdot P - M = \sqrt{m^2 + \vec{q}^{\,2}_{CM}} = (s - M^2 + m^2)/2\sqrt{s} \equiv \omega$, the pion energy, we find

$$f^{3/2}_{3/2,1}(\omega) = -\frac{1}{24\pi}(\omega^2 - m^2)A(\omega) \tag{13}$$

where $A(\omega) \equiv A(\omega,\omega)$ for on-shell scattering. The function B enters in the P_{31} pion-nucleon scattering amplitude, and we will discuss it elsewhere [19]. Elastic unitarity, Eq. (6), applied to the function $A(\omega)$ requires for $\omega > m$

$$\mathrm{Im}A^{-1}(\omega) = \frac{1}{24\pi}(\omega^2 - m^2)^{\frac{3}{2}} \tag{14}$$

From Eq. (11), the on-shell function $A(\omega)$ satisfies, for $\omega > m$,

$$\begin{aligned} A(\omega) \quad &= \quad -2\left(\frac{g_A}{f}\right)^2 \frac{1}{\omega} + \frac{1}{3}\left(\frac{g_A}{f}\right)^2 \mathrm{i}\int \frac{d^4k}{(2\pi)^4}\frac{k^2 - (v\cdot k)^2}{k^2 - m^2 + \mathrm{i}\epsilon} \\ &\times \quad \frac{1}{\omega - v\cdot k + \mathrm{i}\epsilon}\,\frac{A(v\cdot k, \omega\,)}{v\cdot k - \mathrm{i}\epsilon} \end{aligned} \tag{15}$$

The function $A(\omega)$ above develops an imaginary part from two difference sources: the elastic channel and the inelastic channel corresponding to $\pi N \to \pi\pi N$ process, as can be seen in Fig. (2). To obtain, Eq. (14) we have completely neglected all possible inelasticities. We are going to proof that the solution of the Eq. (15) automatically satisfies the elastic unitarity condition, given in Eq. (14), provided the pion energy, ω, is such that we are below the two pion threshold or, equivalently, if the latter contribution to $\mathrm{Im}A(\omega)$ is neglected. This can be easily seen by standard operator methods by writing Eq. (11) in an operator form

$$A(\omega,\omega') \quad = \quad U(\omega,\omega') + \int d\nu\, U(\omega,\nu)\, G(\nu)\, A(\nu,\omega') \tag{16}$$

with an obvious identification of the kernel U and $G(\nu)$ given by

$$G(\nu) = -\frac{i}{6}\int \frac{d^4k}{(2\pi)^4}\frac{k^2-(v\cdot k)^2}{k^2-m^2+\mathrm{i}\epsilon}\frac{1}{\alpha - v\cdot k+\mathrm{i}\epsilon}\delta(\nu - v\cdot k) \tag{17}$$

with $\alpha = v\cdot P - M$. If the inelastic $\pi N \to \pi\pi N$ channel is not considered, the potential $U(\omega,\omega')$ can be considered to be real, so that the corresponding $\mathrm{i}\epsilon$ can be ignored. Thus, one gets

$$A(\omega,\omega') - A^*(\omega',\omega) = \int d\nu\, A^*(\omega',\nu)\, 2\mathrm{i}\mathrm{Im}G(\nu)A(\omega,\nu) \tag{18}$$

By direct application of Cutkosky's rules, we get from Eq. (17)

$$\begin{aligned} 2\mathrm{i}\mathrm{Im}G(\nu) &= -\frac{i}{6}\int \frac{d^4k}{(2\pi)^4}\left[k^2-(v\cdot k)^2\right](-2\mathrm{i}\pi)^2\delta^+(k^2-m^2)\,\delta(\alpha - v\cdot k)\,\delta(\nu - v\cdot k) \\ &= \frac{1}{6}(m^2-\alpha^2)\delta(\alpha-\nu)2\mathrm{i}\mathrm{Im}J_0(\alpha) \end{aligned} \tag{19}$$

where $\delta^+(k^2-m^2) = \theta(k^0)\delta(k^2-m^2)$ and the one-loop function

$$J_0(\alpha) = -\mathrm{i}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2+\mathrm{i}\epsilon}\frac{1}{\alpha - v\cdot k+\mathrm{i}\epsilon} \tag{20}$$

has been defined. It is linearly divergent and hence two subtractions are needed to make it convergent [7],

$$J_0(\alpha) = K_0 + K_1\alpha + \bar{J}_0(\alpha) \tag{21}$$

$$\bar{J}_0(\alpha) = -\frac{\sqrt{\alpha^2-m^2}}{4\pi^2}\{\mathrm{arcosh}\frac{\alpha}{m} - \mathrm{i}\pi\}; \qquad \alpha > m \tag{22}$$

where K_0 and K_1 are related to the divergent constants $J_0(0)$ and $J_0'(0)$. Finally, in the infinite nucleon mass limit, $\omega = \omega' = \alpha$, and Eq.(18) reduces to the unitarity condition of Eq.(14).

An exact solution of the BSE (15) contains both elastic and two pion inelastic contributions, even though Eq. (15) constitutes a lowest order approximation. This feature makes a practical solution of that equation very difficult, mainly due to the non-locality in the momentum variable of the exchanged nucleon. On the other hand, we have seen that the solution of Eq. (15) exactly satisfies the unitarity condition of Eq.(14) when inelasticities are neglected. Besides, the inelastic contributions start at $\mathcal{O}(1/f^4)$, but they are not complete because terms of the same order have not even been included in the iterated *potential* and *propagator* (vertex correction and self-energy insertion). Thus, we propose to solve the BSE (15) in the limit in which the inelastic $\pi N \to \pi\pi N$ channel does not contribute to the imaginary part of $A(\omega)$, although this channel may contribute to the real part of $A(\omega)$ as a subthreshold effect. To do that, we make use of the analytical structure of $A^{-1}(\omega)$ in the complex ω plane implied by the BSE to the lowest order considered. Once that the two pion channel contribution to $\mathrm{Im}A(\omega)$ is neglected, $A^{-1}(\omega)$

only has a right hand cut, with branch points in $\omega = m$ and infinity , and a discontinuity through the cut determined by Eq. (14),

$$A^{-1}(\omega + \mathrm{i}\epsilon) - A^{-1}(\omega - \mathrm{i}\epsilon) = 2\mathrm{i}\mathrm{Im}A(\omega + \mathrm{i}\epsilon) = \frac{\mathrm{i}}{12\pi}(\omega^2 - m^2)^{\frac{3}{2}} \tag{23}$$

On the other hand $A^{-1}(\omega)$ is not expected to have poles which would correspond to zeros[7] of $A(\omega)$ and thus of the cross section. Under these circumstances and as demanded by the asymtotic behaviour deduced from Eq. (23), , a four-times subtracted dispersion relation for $A^{-1}(\omega)$ is needed, which then reads[8] for $\omega > m$:

$$\begin{aligned} A^{-1}(\omega) &= \frac{-f^2\omega}{2g_A^2} + P(\omega) + (\omega^2 - m^2)\bar{J}_0(\omega)/6 \\ P(\omega) &= m^3\left(c_0 + c_1(\frac{\omega}{m} - 1) + c_2(\frac{\omega}{m} - 1)^2 + c_3(\frac{\omega}{m} - 1)^3\right) \end{aligned} \tag{24}$$

The coefficients c_i might be expanded in powers of $1/f^2$. In the former equation we have explicitly separated the lowest Born term $(-\omega f^2/2g_A^2)$ which only affects to the linear coefficient in ω. Thus, we have $c_{0,1,2,3} = \mathcal{O}(1)$ in the infinite nucleon mass limit. We would like to make a few remarks:

- The structure of the solution, Eq. (24), of Eq. (15), agrees with the findings of Ref. [18], where it was shown that in the case of ChPT in the meson-meson sector, to get the on-shell amplitude, the off-shellness of the BSE could be ignored, as long as a renormalized *potential* is iterated. With this philosophy in mind, Eq. (15) becomes

$$\begin{aligned} A^{-1}(\omega) &= H(\omega) - \frac{\omega f^2}{2g_A^2} \\ H(\omega) &= \frac{\mathrm{i}}{6}\int \frac{d^4k}{(2\pi)^4}\frac{k^2 - (v\cdot k)^2}{k^2 - m^2 + \mathrm{i}\epsilon}\frac{1}{\omega - v\cdot k + \mathrm{i}\epsilon} \end{aligned} \tag{25}$$

  The divergent Integral $H(\omega)$ is related to $J_0(\omega)$, $H(\omega) = (\omega^2 - m^2)J_0(\omega)/6 + P_1(\omega)$, with $P_1(\omega)$ a polynomial in the variable ω of degree one. The above equation, is equivalent to Eq. (24). Note, that in this approximation, the nucleon propagator which enters in the iterated *potential*, see Eq. (7), has been taken out of the integral and hence it can never be put on the mass-shell. This is the reason why the approximate solution given in Eq. (25) does not contain the inelastic channel $N\pi\pi$ contribution (see Fig. 2) and thus it can coincide with the solution of Eq. (24), based only on the elastic unitarity requirement.

- If one is going to neglect explicitly the inelastic channel, then only the nucleon propagator $1/(\omega - v\cdot k + \mathrm{i}\epsilon)$ contributes to the elastic cut. This suggests to make

[7] Note that the kinematical zero at $\omega = m$ has been factorized out

[8] Note that, in this case, $A^{-1}(\omega)$ and $(\omega^2 - m^2)J_0(\omega)/6$ have the same analytical structure, and therefore they can only differ by a polynomial in ω.

a Taylor expansion around ω in the integral of Eq. (15), it is to say

$$\frac{k^2-(v\cdot k)^2}{k^2-m^2+\mathrm{i}\epsilon}\,\frac{A(v\cdot k,\omega)}{v\cdot k-\mathrm{i}\epsilon} \;=\; \frac{k^2-\omega^2}{k^2-m^2+\mathrm{i}\epsilon}\,\frac{A(\omega)}{\omega}+\mathcal{O}(v\cdot k-\omega) \tag{26}$$

The leading order of this expansion yields again to Eq. (24).

4 Numerical Results

To prove the usefulness of our main result, Eq. (24), we perform a direct χ^2 fit to the experimental phase shift in the P_{33} channel [20] from threshold up to $\sqrt{s}$ =1500 MeV. We have assigned a 3% uncertainty to the phase shift as in [10] plus a systematic error of one degree to the data (similar treatments are followed in [13] and [15]).

The χ^2 fit yields the following numerical values for the parameters:

$$c_0 = 0.045 \pm 0.021 \qquad c_1 = 0.29 \pm 0.08$$

$$c_2 = -0.17 \pm 0.09 \qquad c_3 = 0.16 \pm 0.03, \tag{27}$$

with χ^2/d.o.f. = 0.2. The description of the data from threshold to the region well above the resonance is pretty good as can be clearly seen in Fig. 3 (upper solid line).

On the other hand, the above coefficients can also be obtained from the $\mathcal{O}(1/f^4)$ HBChPT pieces obtained in Refs. [9] and [10]. Matching the BSE scattering amplitude to that obtained in either of the above references, and in the infinite nucleon mass limit, we find

$$P(\omega) \;=\; \frac{6\pi m^2\omega^2}{(\omega^2-m^2)g_A^4}\times\left(mt^{(3,3)}-4\pi\bar{J}_0(\omega)[t^{(1,1)}]^2\right)\Big|_{\text{Pol. of degree 3 in }\;(\omega-m)} \tag{28}$$

where the dimensionless amplitudes $t^{(3,3)}$ and $t^{(1,1)}$ are defined in Ref. [15]. They depend only on ω/m, g_A and the Low Energy Constants (LEC's):$\tilde{b}_1+\tilde{b}_2$, $b_{16}-\tilde{b}_{15}$ and b_{19} in the notation of Ref. [9]. The right hand side in Eq. (28) is not a polynomial by itself, because it contains chiral logs stemming from the left hand cut, and it has to be Taylor expanded around $\omega = m$ to comply with the polynomial structure of Eq. (24).

Using the values obtained in Ref. [10] for the LEC's and translated into the notation of [9] as shown in Table I of Ref. [15] (Set **II** entry) we find after Montecarlo propagation of errors, the lower solid and dash–dotted lines in Fig. 3 and estimates for the coefficients $c_{0,1,2,3}$

$$c_0^{\rm th} = 0.001 \pm 0.003 \qquad c_1^{\rm th} = 0.038 \pm 0.006$$

$$c_2^{\rm th} = 0.064 \pm 0.005 \qquad c_3^{\rm th} = 0.036 \pm 0.002 \tag{29}$$

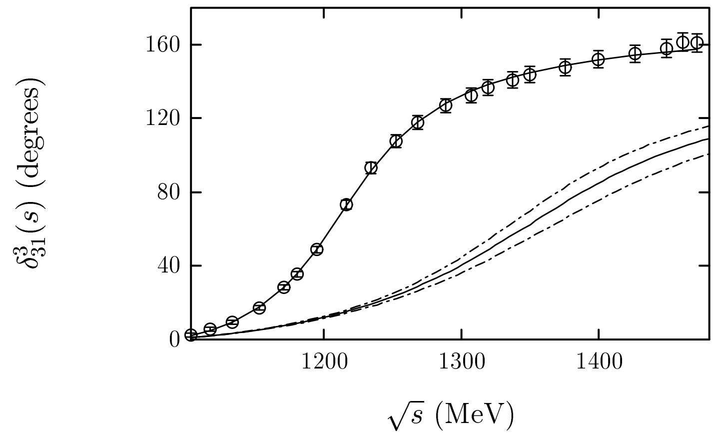

Figure 3: P_{33} phase shifts as a function of the total CM energy $\sqrt{s}$. The upper solid line represents a χ^2-fit of the parameters of Eq. (24) to data of Ref. [20] (circles). Best fit parameters are given in Eq. (27). The lower lines stand for the results obtained with the parameters given in Eq. (29). Central values lead to the solid line, whereas the errors on Eq. (29) lead to the dash-dotted lines.

Obviously, the structure of the Eq. (24) which ensures exact elastic unitarity, allows for a satisfactory fit to data, as long as the contributions coming from two pion production are small and the left-hand-cut contribution can be modeled, in the scattering region, by a polynomial of the third degree in the variable $\omega - m$.

In the extreme static approximation, the HBChPT estimate of the polynomial $P(\omega)$ provides the bulk of the resonant shape, although it is far from describing accurately the data. Similar conclusions, within the IAM scheme, were reached in Ref. [15]. There also, it was shown that the $1/Mf^2$ corrections (NLO in the chiral expansion) are extremely important. This might explain the big discrepancies between the set of parameters of Eqs. (27) and (29). Results in Eq. (27) are obtained from a best fit to data and effectively incorporate, buried in the fitted parameters, all higher order corrections, in particular the above mentioned $1/Mf^2$ pieces.

5 Conclusions

In this paper we have dealt with the study of the $\Delta(1232)$ resonance as it arises in πN scattering in the P_{33} channel. We have used HBChPT as a perturbative guide to implement in a sensible way the constraints imposed by chiral symmetry. The Bethe-Salpeter equation provides, in addition, a workable scheme where unitarity is exactly

restored up to the level imposed by the corresponding kernel and, moreover, allows for an identification of the infinite set of diagrams being summed up. In the case of πN scattering in the P_{33} channel we have used HBChPT to lowest order as input. The BSE is able, out of its divergence structure, to generate some of the higher order counter-terms required in standard HBChPT in the form of subtraction constants for the divergent integrals. Actually, our BSE amplitude admits a very good fit to the experimental phase shifts in terms of four subtraction constants. Nevertheless, their fitted values differ, at least within estimated errors, from those deduced from HBChPT. The discrepancy may be understood because the NLO $(1/Mf^2)$ contributions are comparable to the LO $(1/f^2)$ ones, and hence the matching procedure of the BSE solution to standard HBChPT amplitudes induces large numerical corrections. This situation is not new and was already encountered in previous studies based on the IAM [14, 15] which require a similar matching procedure of unitarized amplitudes to the standard ones of HBChPT. As we see, however, the amount of work required to describe the πN scattering data within the BSE, and in particular, the $\Delta(1232)$ resonance is drastically smaller than that needed for the IAM. In addition, as we pointed out in Ref. [18] for the meson-meson problem, one can search for systematic improvement and convergence in a BSE framework. A full study of the πN system along these lines is left for future research [19].

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